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Thermally Developing Laminar Flow in a Duct with External Radiation and Convection

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Nomenclature

B_i = Biot number, $\frac{1}{2}Nu_o$
h_{xi} = local internal heat-transfer coefficient
h_o = external heat-transfer coefficient
k_i = thermal conductivity of internal flow
k_o = thermal conductivity of external flow
m = $(n+1)/n$
n = flow-type parameter, $n=1$ for Newtonian flow
Nr = radiation parameter, $\sigma\epsilon Tr^3 R/k_o$
Nu_o = Nusselt number, $2h_oR/k_o$
Pe = Peclet number of inner fluid, $2\bar{u}R/\alpha$
q_o'' = external heat flux
Q_o = dimensionless external heat flux, $q_o''R/k_oT_r$
r = radial coordinate or direction normal to axial axis
R = radius for circular conduit or half-width for flat channel
S = geometry parameter, $S=0$ for flat channel, $S=1$ for pipe
T = temperature
u = velocity of the internal or inner fluid
$\bar{u}$ = mean velocity of inner fluid
x = axial coordinate
X = dimensionless axial coordinate, $(x/R)/Pe$
y = distance from inner wall
α = thermal diffusivity
δ = thermal boundary-layer thickness
ϵ = emissivity of outer wall
σ = Stefan-Boltzmann constant

Subscripts

i = inlet condition for internal flow
o = external surface
r = reference point
w = wall condition
x = axial dependent
∞ = external environment or ambient condition

Introduction

THE classical problem of laminar thermal entrance flow in a circular tube is that in which either the wall temperature or the heat flux at the wall is prescribed.[1] Even in a more complicated case that involves convective heat exchange between the outer surface of the conduit and a fluid environment, it is usually assumed that the value of the external heat-transfer coefficient is known a priori and is normally taken to be a constant. In reality, the external heat-transfer coefficient often depends on the wall temperature, which is an unknown.

The problem of laminar forced convection in pipe flow subjected to thermal radiation has received considerable attention.[2-6] The majority of previous investigations on this subject is limited to the case with the environment temperature at absolute zero. More recently, Faghri and Sparrow[7] applied a numerical scheme developed earlier by Patankar and Spalding to solve the problem of laminar flow in a horizontal pipe subjected to external natural convection and radiation.

This Note considers the same problem presented in Ref. 7, but uses a simpler approach, the heat balance integral method. However, the present analysis is applied equally to the power law fluids (non-Newtonian flow), which none of the previous investigators has considered. Flow inside a flat conduit is also included in this analysis.

Formulation of the Problem

Consideration is given to a laminar flow with constant physical properties and with a uniform temperature T_i entering a channel in which the flow is fully developed hydrodynamically but is developing thermally. It is assumed that the heat is transferred from the inside channel surface by convection and conduction to the fluid and from the outside channel wall by radiation and forced or natural convection to an external environment at temperature T_∞. In addition, the channel is subjected to an external uniform heat flux or an internal uniform generation within the wall. If the viscous dissipation and the wall resistance are negligible, the steady-state temperature field can be described by the following mathematical expression:

$$\frac{1}{r^S}\frac{\partial}{\partial r}\left(r^S\frac{\partial T}{\partial r}\right)=\frac{u}{\alpha}\frac{\partial T}{\partial x} \tag{1}$$

Presented as Paper 83-0529 at the AIAA 21st Aerospace Sciences Meeting, Reno, Nev., Jan. 10-13, 1983; submitted April 9, 1983; revision submitted July 19, 1983.

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Table 1 Local Nusselt number, Nu_{xi} for Newtonian and non-Newtonian fluids with $Nr=1$ and $S=1$

X	$n=1$	$n=0.5$	$n=0.25$	X	$n=1$	$n=0.5$	$n=0.25$
0.001	15.38	16.77	17.96	0.01	7.98	8.45	9.11
0.002	12.70	13.68	14.76	0.02	6.68	7.05	7.55
0.003	11.29	12.10	13.08	0.03	6.11	6.43	7.86
0.004	10.38	11.08	11.98	0.04	5.78	6.07	6.46
0.005	9.72	10.36	11.20	0.05	5.55	5.83	6.20
0.006	9.22	9.81	10.60	0.06	5.36	5.64	6.00
0.007	8.82	9.37	10.12	0.07	5.16	5.45	5.81
0.008	8.49	9.01	9.72	0.08	4.88	5.19	5.60
0.009	8.21	8.71	9.39	0.09	4.43	4.68	5.28
				0.10	3.43	3.60	4.07

Note: $u=\frac{3+s}{4}\left(\frac{1+3n}{1+n}\bar{u}\right)\left[1-\left(\frac{r}{R}\right)^m\right]$ where $m=\frac{n+1}{n}$.

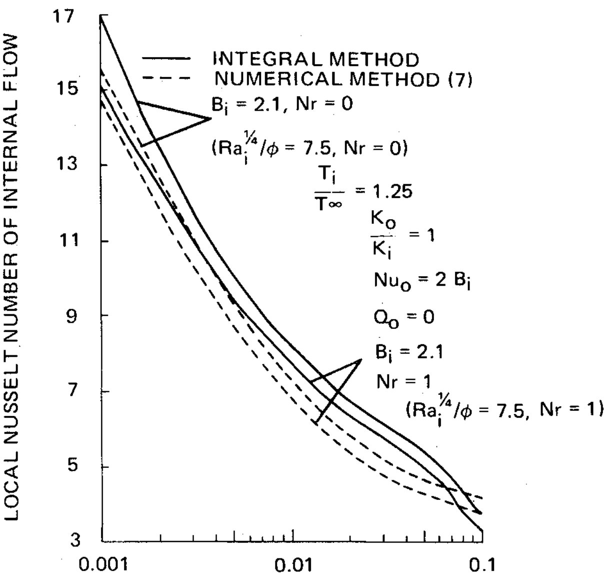

Fig. 1 Variation of local Nusselt number.

where

$$u=\frac{3+s}{4}\left(\frac{1+3n}{1+n}\bar{u}\right)\left[1-\left(\frac{r}{R}\right)^{(n+1)/n}\right] \tag{2}$$

The associated boundary conditions are as follows:

$$T=T_i \qquad \text{at } x=0 \tag{3a}$$

$$\frac{\partial T}{\partial y}=-\frac{\partial T}{\partial r}=0 \qquad \text{at } r=0 \,(\text{or } y=R) \tag{3b}$$

$$k_i\frac{\partial T}{\partial y}=-k_i\frac{\partial T}{\partial r}=h_o(T_{wx}-T_\infty)$$

$$+\sigma\epsilon(T_{wx}^4-T_\infty^4)-q_o'' \qquad \text{at } r=R \text{ (or } y=0) \tag{3c}$$

where $y=R-r$.

The circumferential average value of external heat-transfer coefficient h_o is, in general, a function of wall temperature T_{wx}, which is in terms of distance x from the inlet of the channel in the thermally developing region. In the case of natural convection, h_o becomes a power law of the wall temperature. Because of nonlinear conditions [Eq. (3c)], the exact analytical solution is not feasible. The thermal boundary-layer concept is used to define a thermal boundary-layer thickness δ. For the region beyond the thermal boundary layer, the inner fluid temperature is equal to the inlet temperature T_i. Therefore, the following conditions are also valid:

$$\frac{\partial T}{\partial y}=-\frac{\partial T}{\partial r}=0 \qquad \text{at } y\geq\delta \text{ (or } r\leq R-\delta) \tag{3d}$$

and

$$T=T_i \qquad \text{at } y\geq\delta \text{ (or } r\leq R-\delta) \tag{3e}$$

The detailed analysis is contained in Ref. 8.

Results and Discussions

To demonstrate the validity of the present solution, various comparisons have been made in Ref. 8. Good agreement was found between the present and previous solutions for a Newtonian laminar tube flow with uniform wall heat flux, radiation to an environment with the absolute zero temperature, and convection and radiation at the external surface. Only the last case is presented here. To examine this case, the numerical examples for constant external heat-transfer coefficient with or without thermal radiation are presented in Fig. 1. The finite difference solutions of Faghri and Sparrow[7] are also shown in the figure. The agreement between the two solutions is reasonably good. Similar trends are found for a more general case in which external heat-transfer coefficients are functions of temperature, such as natural convection. The same natural convection coefficient used in Ref. 7 is adopted here, i.e.,

$$Nu_o=0.36+0.518Ra_i^{1/4}\left(\frac{T_{wx}-T_\infty}{T_i-T_\infty}\right)^{1/4}\Big/\phi \tag{4}$$

where

$$\phi=[1+(0.559/P_r)^{9/16}]^{4/9} \tag{5a}$$

$$Ra_i=g\beta(T_i-T_\infty)(2R)^3/\alpha\nu \tag{5b}$$

Even without including thermal radiation, Eq. (4) represents a nonlinear boundary condition. The results for the case involving natural convection with or without radiation are also included in Fig. 1. Note that the curves labeled $Ra_i^{1/4}/\phi=7.5$ coincide with those labeled $B_i=2.1$.

It is also of interest to examine a more complicated situation involving non-Newtonian fluid ($n\neq1$) in a circular pipe ($S=1$) and thermal radiation at the outer surface. To our knowledge, no previous solution exists for this case. Table 1 presents the local Nusselt number at various locations with radiation parameter equal to unity. For comparison, the corresponding Newtonian flow under the same thermal and flow conditions is included. Note that the Nusselt number increases as the parameter n decreases. Although no experimental data are available to justify this finding, the similar trend is also found in a recent theoretical study of non-Newtonian flow with a prescribed surface temperature at the wall.[9]

There has always been a question of choice of temperature profiles in applying the integral method to solving heat-transfer problems. Özisik[10] has made a comment for the case of pure heat conduction that the results are rather insensitive as to choice of form of the profile; a choice of higher order may improve the result slightly but adds more algebraic complexity. For practical purposes, there is little gain in accuracy by using a profile higher than fourth degree. Recently, Yeh and Chung[11] also indicated that the choice of a higher-order profile in a more general problem, combined conduction and radiation in a semi-infinite solid, improves the temperature profile slightly at y near δ but has little effect on the heat-transfer rate at the wall. Therefore, a choice of higher order temperature profile in the present analysis may not be worthwhile.

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